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Investigation of temperature anisotropy in highly-magnetized plasma

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Abstract

We investigate the temperature anisotropy in highly-magnetized plasma within the framework of kinetic theory. We explicitly calculate the electronic distribution function for a magnetized plasma, taking into account electron-ion (e-i) collisions. The basic equation in this investigation is the Fokker-Planck (F-P) equation, where some justified approximations for fusion and astrophysical magnetized plasmas are used. By computing the second moment of this distribution function, we have expressed the electron temperatures in the parallel direction as well as in the plane perpendicular to the magnetic field. We show that the temperature is anisotropic and that this anisotropy is due to a competition between the magnetic field and the collision effects. We also present the numerical results and interpret them for illustration. Our theoretical analysis is applicable in wave and instability studies in fusion and astrophysical plasma, particularly in magnetized inertial fusion (MIF) scheme.

Keywords: magnetic thermonuclear fusion; magnetized plasma; plasma kinetic theory; Coulomb collision in plasma

1. Introduction

A magnetized plasma is one in which an ambient magnetic field is strong enough to significantly alter particle trajectories. This kind of plasma is a good environment for different physical phenomena which have intensively been studied in literature, namely, Alfvén wave [1,2], cyclotron instabilities [3], and magnetic field reconnection [4,5] .

Magnetized plasma, both in astrophysical medium or that created in laboratories, generally presents an anisotropy in temperature [6] which can be interpreted in the microscopic way by an anisotropic distribution function.

In the literature, this distribution function is usually assumed to be a bi-Maxwellian distribution function:

$$f_{BM}(v_{\parallel}, v_{\perp}) = \frac{n_e}{T_{\perp} T_{\parallel}^{\frac{1}{2}}} \exp\left(-\frac{m_e v_{\parallel}^2}{2T_{\parallel}}\right) \exp\left(-\frac{m_e v_{\perp}^2}{2T_{\perp}}\right), \quad (1)$$

where m_e, n_e, $T_{\parallel}$, $T_{\perp}$, $v_{\parallel}$ and $v_{\perp}$ are respectively the electron mass, the electronic density, the parallel temperature, the perpendicular temperature, the parallel velocity and the perpendicular velocity.

The aim of the present paper is to analyze the electron temperature anisotropy for magnetized plasma, in the frame of the kinetic theory. This investigation could have applications in several research axes, such as magnetic fusion experiments [7,8].

The magnetized plasma appears at the microscopic level as a set of charged particles of different species in thermal motion at different velocities, where each particle has a fast gyration motion around the magnetic field line at a perpendicular velocity $v_{\perp}$, and a parallel motion not affected by the magnetic field. The time dependent electron velocity can be written as:

$\vec{v}(t) = \vec{v}_{\parallel} + \vec{v}_{\perp}(t)$, where $\vec{v}_{\perp}(t)$ is the time varying perpendicular velocity which is proportional to $\exp(i\omega_{ce} t)$, where $\omega_{ce} = \frac{eB}{m_e}$ is the electron cyclotron frequency and B is the applied magnetic field . Note here that ω_{ce} is the same for all electrons in the plasma [9]. In order to compute the electronic distribution function, we consider for the one particle kinetic theory in 6D phase space: $(\vec{r}, \vec{v})$. The Fokker Planck (F-P) equation is then the appropriate equation for describing these kinds of plasmas [10], where the distribution depends on the three independent parameters: $v_{\parallel}, v_{\perp}$ and the time t.

In the present investigation, we consider that the time evolution of the electron distribution function is characterized by two time scales as was the case in our previous works [11-15]: a short time scale relative to the cyclotron motion of electrons around the magnetic field lines, $\tau_{ce} = \frac{1}{\omega_{ce}}$ (which has typical values of $\tau_{ce} \sim 10^{-11}\ s$ for magnetic thermonuclear fusion experiments, where $\omega_{ce} \sim 10^{11}\ s^{-1}$) and a relatively long hydrodynamic time scale ($\tau_{hy} \gg \tau_{ce}$).

This paper is organized as follows: in section 2, we present the basic equation used in this investigation. In section 3, the equation of the distribution function is analytically calculated under some justified approximations. In section 4, we compute the high frequency distribution function. In section 5, we compute the static distribution function. In section 6, we compute the parallel temperature and the perpendicular one, where the anisotropy in temperature is explicitly presented. Finally, in section 7, a conclusion is given for the obtained results.

2. Basic equation

The basic equation in this investigation is the Fokker-Planck (F-P) equation. The F-P equation can be presented for a homogeneous plasma, in the presence of the Lorentz force due to a statistic magnetic field, $\vec{F}_L(t) = -e\vec{v}(t) \times \vec{B}$, taking into account the e-i Coulomb collisions, following the Braginskii notation [16,17] as follows:

$$\frac{\partial f}{\partial t} + \frac{\vec{F}_L}{m_e} . \frac{\partial f}{\partial \vec{v}} = C_{ei}(f), \qquad (2)$$

where $f = f(\vec{v}, \vec{r}, t)$ is the electrons distribution function and $C_{ei}(f)$ represents the e-i operator. Note here that the distribution function depends on the three independent parameters ($v_{\parallel}$, $v_{\perp}$ and t) and the Lorentz force is a time dependent force.

Without loss of generality we consider the magnetic field to be oriented in the x direction, $\vec{B}$=$B\hat{x}$, and the electrons to oscillate in the (y, z) plane, where: $\overrightarrow{v_{\perp}}(t) = v_{\perp}(\hat{z} - i\hat{y}) \exp(i\omega_{ce} t)$.With this geometry, the Lorentz force is given by:

$$\vec{F}_L = -m_e \omega_{ce} v_{\perp}(\hat{y} + i\hat{z}) \exp(i\omega_{ce} t). \qquad (3)$$

This force is similar to that due to the presence of a circularly-polarized laser wave in the plasma [11]. Taking Eq. (3) into account, the F-P equation (Eq. 2) is written as:

$$\frac{\partial f}{\partial t} - \omega_{ce} v_{\perp} \left(\frac{\partial f}{\partial v_y} + i \frac{\partial f}{\partial v_z} \right) \exp(i\omega_{ce} t) = C_{ei}(f).$$

(4)

We point out that this equation (Eq. 4) is similar to that which characterizes a homogenous plasma in interaction with a circularly polarized laser wave [11, 13]. Then we be expecting an anisotropy in temperature due to the presence of magnetic field.

3. Distribution function

The motion of individual charged particle in plasma, in the presence of a static magnetic field, can be decomposed into a parallel motion not affected by the magnetic field and a perpendicular gyration motion.

The gyration period time is typically very small compared to the hydrodynamic evolution time of the plasma. Then it is judicious to separate the time scales in the F-P equation, (Eq. 4), by assuming that the distribution function is the sum of oscillating distribution function and a static one relative to evolution of hydrodynamic parameters in the plasma. Hence, we write:

$$f = f(v_{\parallel}, v_{\perp}, t) = f^s(v_{\parallel}, v_{\perp}, t) + \text{Real}\{ f^h(\vec{v}, t)\} \quad ,$$

(5)

$$f^h(\vec{v}, t) = f^h(v_{\parallel}, v_{\perp}) \exp(i\omega_{ce} t). \qquad (6)$$

The separation of time scales in the F-P equation, (Eq. 4), using Eq. (5), gives rise to a system of two coupled equations: a fast time variation equation which represents the spatiotemporal evolution of f^h and a slow time variation equation representing the spatiotemporal evolution of f^s. Thus:

$$\frac{\partial f^h}{\partial t} - \omega_{ce} v_{\perp} \left(\frac{\partial f^s}{\partial v_y} + i \frac{\partial f^s}{\partial v_z} \right) \exp(i\omega_{ce} t) = C_{ei}(f^h), \qquad (7)$$

This equation is obtained by regrouping the fast time-varying terms, proportional to $\exp(i\omega_{ce} t)$, in Eq. (4).

The equation of the static distribution function is obtained by taking the average of Eq. (4) on the cyclotron period, $\tau_{ce} = \frac{2\pi}{\omega_{ce}}$, so:

$$\frac{\partial f^s}{\partial t} - \omega_{ce} v_\perp \left\langle \begin{matrix} \mathrm{Real}(\exp(i\omega_{ce} t)) \times \\ \mathrm{Real}\left(\frac{\partial f^h}{\partial v_y} + i\frac{\partial f^h}{\partial v_z}\right) \end{matrix} \right\rangle_{\tau_{ce}} = C_{ei}(f^s). \quad (8)$$

Here the symbol $\langle X \rangle_{\tau_{ce}} = \frac{1}{\tau_{ce}} \int_0^{\tau_{ce}} X dt$ stands for the average value over the cyclotron period time.

4. **High-frequency distribution function**

Using expression (6), f^h can be calculated from equation (7), where $\frac{\partial f^h}{\partial t} = i\omega_{ce} f^h$, as a function of f^s. Thus:

$$i\omega_{ce} f^h - C_{ei}(f^h) = \omega_{ce} v_\perp \left(\frac{\partial f^s}{\partial v_y} + i\frac{\partial f^s}{\partial v_z}\right) \exp(i\omega_{ce} t). \quad (9)$$

The collision operator, $C_{ei}(f)$, is expressed in Landau form of the F-P collision operator [18,19,20] as:

$$C_{ei}(f) = \frac{A}{v^3} \frac{\partial}{\partial v_j} \left(v_j v_k - v^2 \delta_{jk}\right) \frac{\partial f}{\partial v_k}, \quad (10)$$

where $A = \frac{v_t^4}{2\lambda_{ei}}$, $\lambda_{ei} = \frac{4\pi\varepsilon_0 T_e{}^2}{n_e e^4 Z \ln\Lambda}$ is the mean free path, $\nu_{ei} = \frac{1}{2}\frac{v_t}{\lambda_{ei}}$ and $v_t = \sqrt{T_e/m_e}$ is the thermal velocity. Note that we used Einstein's notation in equation (10).

The e-i collision operator (10) has the spherical-harmonics like proper functions [21-23]. Then it is judicious to use the spherical system $(v, \mu = \frac{v_x}{v}, \varphi = \mathrm{arctg}\frac{v_y}{v_z})$. The right hand side of equation (9) is written then as:

$$\omega_{ce}\left((1-\mu^2)^{3/2}\left(v\frac{\partial f^s}{\partial v} + \mu\frac{\partial f^s}{\partial \mu}\right)\right) \times \exp(i\omega_{ce} t + i\varphi) \quad . \quad (11)$$

This shows that f^h is proportional to $\exp(i\varphi)$ and f^s is independent of φ. It is therefore practical to expand $f^s(\vec{v}) = f^s(\mu, v)$ in Legendre polynomials, $P_l(\mu)$: $f^s = \sum P_l(\mu) f_l^s(v)$, and to expand the function $f^h = f^h(\mu, v) exp(\omega_{ce} t + \varphi)$, in spherical harmonics, $Y_l^1(\mu, \varphi)$,of order $(l, m = 1)$:

$$f^h = \sum_{l=0}^{l=\infty} Y_l^1(\mu, \varphi) f_l^h(v) = \exp(i\varphi) \sum_{l=0}^{l=\infty} P_l^1(\mu) f_l^h(v),$$

where $P_l^1(\mu)$ is the associated Legendre polynomial of *order* $(l, m = 1)$. Considering these expansions, the high frequency equation, (9), can be written as:

$$\left(i\omega_{ce} + l(l+1)\frac{A}{v^3}\right) \sum_{l=0}^{l=\infty} P_l^1 f_l^h(v) = - \omega_{ce}\left\{(1-\mu^2)^{3/2}\left(\begin{matrix} v\sum_{l=0}^{l=\infty} P_l \frac{\partial f_l^s}{\partial v} + \\ \mu\sum_{l=0}^{l=\infty} \frac{\partial P_l}{\partial \mu} f_l^s \end{matrix}\right)\right\}. \quad (12)$$

After some algebra using recurrence relations between Legendre polynomials and associated Legendre polynomials [21],we have explicitly calculated the f_l^h as functions of $f_{l-3}^s, f_{l-1}^s, f_l^s, f_{l+1}^s$ *and* f_{l+3}^s, hence:

$$f_l^h = [G_1(l) v\frac{\partial f_{l-3}^s}{\partial v} + G_2(l) v\frac{\partial f_{l-1}^s}{\partial v} + G_3(l) v\frac{\partial f_{l+1}^s}{\partial v} + G_4(l) v\frac{\partial f_{l+3}^s}{\partial v} + G_5(l) f_{l-3}^s + G_6(l) f_{l-1}^s$$
$$G_7(l) f_{l+1}^s + G_8(l) f_{l+3}^s] i\exp(i\omega_{ce} t + i\varphi), \quad (13)$$

where $G_1(l) = \frac{(l-2)(l-1)}{(2l-5)(2l-3)(2l-1)}$,

$$G_2(l) = -\left(\frac{l(l+1)}{(2l-1)(2l+1)(2l+3)} + \frac{(2l-3)(2l-1)(2l+1) - l^2(2l-3) - (l-1)^2(2l+1)}{(2l-3)(2l-1)^2(2l+1)}\right),$$

$$G_3(l) = \frac{(2l+1)(2l+3)(2l+5) - (l+2)^2(2l+1) - (l+1)^2(2l+5)}{(2l+1)(2l+3)^2(2l+5)} + \frac{(l+1)l}{(2l+3)(2l+1)(2l-1)}, \quad G_4(l) = -\frac{(l+3)(l+2)}{(2l+7)(2l+5)(2l+3)}.$$

$$G_5(l) = \frac{(l-3)(l-2)^2}{(2l-5)(2l-3)(2l-1)}, \quad G_6(l) = -\frac{(2l+3)(l-1)l[(l-2)(2l+1) + (l+1)(2l-3)] + (l-1)l^2(2l-3)(2l-1)}{(2l-3)(2l-1)^2(2l+1)(2l+3)},$$

$$G_7(l) = \frac{(2l-1)(l+1)(l+2)[l(2l+5) - (l+3)(2l+1)] - (l+1)^2(l+2)(2l+3)(2l+5)}{(2l-1)(2l+1)(2l+3)^2(2l+5)}$$

and $G_8(l) = \frac{(l+3)^2(l+4)}{(2l+3)(2l+5)(2l+7)}$.

Note that in this equation, the highly-magnetized plasma approximation ($\omega_{ce} \gg \nu_{ei}$) is used.

5. Static distribution function

The second term in the left-hand side of the static distribution function equation, Eq. (8), can be written using spherical coordinates as:

$$\omega_{ce}\langle \mathrm{Real}(v_{v_\perp} \exp(i\omega_{ce} t) \times \mathrm{Real}(\frac{\partial f^h(v,\mu,\varphi,t)}{\partial v_y} + \frac{\partial f^h(v,\mu,\varphi,t)}{\partial v_z}))\rangle_{\tau_{ce}} =$$
$$\frac{\omega_{ce}}{2}(1-\mu^2)^{3/2} \times$$

$$\left(v\frac{\partial f^h(v,\mu)}{\partial v}-\mu\,\frac{\partial f^h(v,\mu)}{\partial\mu}+\frac{f^h(v,\mu)}{(1-\mu^2)}\right). \quad (14)$$

The equation of the static distribution function is then given in the spherical coordinates by:

$$\frac{\omega_{ce}}{2}(1-\mu^2)^{3/2}\times$$
$$\left\{v\frac{\partial f^h(v,\mu)}{\partial v}-\mu\,\frac{\partial f^h(v,\mu)}{\partial\mu}+\frac{f^h(v,\mu)}{(1-\mu^2)}\right\}$$
$$=\frac{A}{v^3}\left(\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial f^s(v,\mu)}{\partial\mu}\right). \quad (15)$$

We expand, as in the section 4, the $f^s(v,\mu)$ in $P_l(\mu)$ and the $f^h(v,\mu)$ in the $P_l^1(\mu)$, hence:

$$\frac{\omega_{ce}}{2}\sum_{l=0}^{l=\infty}\{v\frac{\partial f_l^h}{\partial v}(1-\mu^2)^{3/2}P_l^1-(1-\mu^2)^{3/2}\mu\frac{\partial P_l^1}{\partial\mu}f_l^h+$$
$$(1-\mu^2)^{1/2}P_l^1f_l^h\}=\frac{A}{v^3}\sum_{l=0}^{l=\infty}l(l+1)P_l\,f_l^s.$$
(16)

After some algebra using recurrence relations between Legendre polynomials, $P_l(\mu)$, and associated Legendre polynomials, $P_l^1(\mu)$, this equation (16) is written as follows:

$$\frac{\omega_{ce}}{2}\ \{G_9(l)v\frac{\partial f_{l-3}^h}{\partial v}+G_{10}(l)v\frac{\partial f_{l-1}^h}{\partial v}+G_{11}(l)v\frac{\partial f_{l+1}^h}{\partial v}+$$
$$G_{12}(l)v\frac{\partial f_{l+3}^h}{\partial v}+$$
$$G_{13}(l)f_{l-3}^h+G_{14}(l)f_{l-1}^h+G_{15}(l)f_{l+1}^h$$
$$+G_{16}(l)f_{l+3}^h\}=\frac{A}{v^3}\,l(l+1)\,f_l^s(v), \quad (17)$$

where $G_9(l)=-\frac{(l-3)(l-2)(l-1)l}{(2l-5)(2l-3)(2l-1)}$, $G_{10}(l)=$

$\frac{(l-1)l(l+1)(l+2)}{(2l-1)(2l+1)(2l+3)}+$

$\frac{(l-1)l[(l-1)(l+1)(2l-3)+l(l-2)(2l+1)+(2l+1)(2l-1)(2l-3)]}{(2l-3)(2l-1)^2(2l+1)}$

$G_{11}(l)=$

$\frac{(l-1)l(l+1)(l+2)}{(2l+3)(2l+1)(2l-1)}-$

$\frac{(l+1)(l+2)[(l+1)(l+3)(2l+1)+(l+2)l(2l+5)+(2l+5)(2l+3)(2l+1)]}{(2l+1)(2l+3)^2(2l+5)}$

$G_{12}(l)=\frac{(l+1)(l+2)(l+3)(l+4)}{(2l+7)(2l+5)(2l+3)}$, $G_{13}(l)=\frac{(l-3)^2(l-2)(l-1)^2l}{(2l-5)(2l-3)(2l-1)}$,

$G_{14}(l)=$

$-\frac{l^3(l-1)^2(l-2)}{(2l-3)(2l-1)^2}-\frac{(l+1)l^2(l-1)^3}{(2l+1)(2l-1)^2}-\frac{(l-1)^2l(l+1)^2(l+2)}{(2l-1)(2l+1)(2l+3)}+$

$\frac{(l-1)l}{(2l-1)}$,

$G_{15}(l)=$

$\frac{(l+2)^3(l+1)^2l}{(2l+1)(2l+3)^2}+\frac{(l+3)(l+2)^2(l+1)^3}{(2l+5)(2l+3)^2}+\frac{(l+2)^2(l+1)l^2(l-1)}{(2l-1)(2l+1)(2l+3)}-$

$\frac{(l+1)(l+2)}{2l+3}$ and $G_{16}(l)=-\frac{(l+4)^2(l+3)(l+2)^2(l+1)}{(2l+3)(2l+5)(2l+7)}$.

This equation coupled with the f_l^h formula, (Eq. 13), allows us to determinate the different components, $f_l^s(v)$, of the static distribution function by knowing f_0^s as a boundary condition. The zeroth-order static distribution function corresponds to the non-perturbed (by the magnetic field) distribution function of electrons. It can then be estimated by considering the thermodynamic equilibrium as a Maxwell function. At this order (zero), the high frequency function vanishes.

More interest is given to the second anisotropy, f_2^s, which is responsible for temperature anisotropy, so:

$$f_2^s=\frac{1}{12}\frac{\omega_{ce}}{\vartheta_{ei}(v)}\ \left(\frac{36}{35}v\frac{\partial f_1^h}{\partial v}-\frac{96}{35}v\frac{\partial f_3^h}{\partial v}-\frac{40}{77}v\frac{\partial f_5^h}{\partial v}-\right.$$
$$\left.\frac{960}{77}f_5^h-\frac{2}{7}f_1^h+\frac{68}{7}f_3^h\right), \quad (18)$$

where $\vartheta_{ei}(v)$ is the velocity-dependent frequency relative to electrons having a velocity v.

Neglecting higher-order components behind the f_0^s component, considering that $f_{l+2}^s\ll f_l^s$, this last equation can be written as:

$$f_2^s=\frac{\omega_{ce}}{12\vartheta_{ei}(v)}\times$$
$$\left(-0.06857v\frac{\partial f_0^s}{\partial v}+0.01904v\frac{\partial}{\partial v}\left(v\frac{\partial f_0^s}{\partial v}\right)\right). \quad (19)$$

6. Temperature anisotropy

By limiting the expansion of the distribution function in Legendre polynomials to second order, the parallel temperature $T_\parallel=\overline{m_e v_\parallel^2}$, where the symbol $\overline{}$ stands for average value, is given by:

$$n_eT_\parallel=m_e\int v_\parallel^2 f d^3\vec{v}=$$
$$\pi m_e\int\mu^2v^4\left\{\begin{matrix}f_0(v)+P_1(\mu)f_1(v)+\\P_2(\mu)f_2(v)\end{matrix}\right\}dvd\mu=$$
$$\frac{4}{3}\pi m_e\int v^4\{f_0(v)\}dv-\frac{8}{15}\pi m_e\int v^4\{f_2(v)\}dv.$$
(20)

It is important to note that the high-frequency distribution function does not contribute to the temperature since its average over the cyclotron period time vanishes $[f^h\sim\exp(i\omega_{ce}t)]$. The zeroth order distribution function corresponding to the plasma not being affected by the magnetic field is considered to be a Maxwellian :

$f_0(v) = \frac{n_e}{v_t^3(2\pi)^{3/2}} \exp\left(-\frac{v^2}{2v_t^2}\right)$. Consequently, the second anisotropic distribution function (Eq. 19), can be written as follow:

$$f_2^s = -\frac{\omega_{ce}}{\nu_{ei}} \frac{n_e}{v_t^3(2\pi)^{3/2}} \times \left(0.011809\frac{v^5}{v_t^5} - 0.0057\frac{v^7}{v_t^7}\right) \exp\left(-\frac{v^2}{2v_t^2}\right). \quad (21)$$

Computing the integral in Eq. (20), the explicit expression of $T_{\parallel}$ is found to be:

$$T_{\parallel} = T\left(1 + a\frac{\omega_{ce}}{\nu_{ei}}\right), \quad (22)$$

where ν_{ei} is the e-i collision frequency and $a \approx 1.93$.

The perpendicular temperature, $T_{\perp} = \frac{1}{2}\overline{m_e v_{\perp}^2}$, is given by:

$$n_e T_{\perp} = \frac{1}{2} m_e \int v_{\perp}^2 f d^3\vec{v} = m_e \int (1-\mu^2) v^4 (f_0 + \mu f_1 + \frac{1}{2}(3\mu^2 - 1) f_2) dv d\mu$$

$$= \frac{4}{3}\pi m_e \int v^4 f_0 dv - \frac{4}{15}\pi m_e \int v^4 f_2 dv. \quad (23)$$

In the case of the Maxwellian isotropic distribution function, the $T_{\perp}$ is calculated explicitly from the above equation to be:

$$T_{\perp} = T\left(1 + \frac{a}{2}\frac{\omega_{ce}}{\nu_{ei}}\right). \quad (24)$$

The temperature anisotropy is then given by:

$$\frac{T_{\parallel}}{T_{\perp}} = \frac{1 + a\frac{\omega_{ce}}{\nu_{ei}}}{1 + \frac{a}{2}\frac{\omega_{ce}}{\nu_{ei}}}. \quad (25)$$

It is very clear that this anisotropy depends on the ratio of the cyclotron frequency to the collision frequency. This equation shows that the anisotropy tends to 1 for a high collision frequency ($\frac{\omega_{ce}}{\nu_{ei}} \ll 1$) which is in agreement with the 1D numerical simulation carried out by Takizuka et al. [24], despite that Eq. (25) is limited to highly magnetized plasma ($\frac{\omega_{ce}}{\nu_{ei}} \gg 1$).

We have presented, on the Fig. 1, the anisotropy on the distribution function.

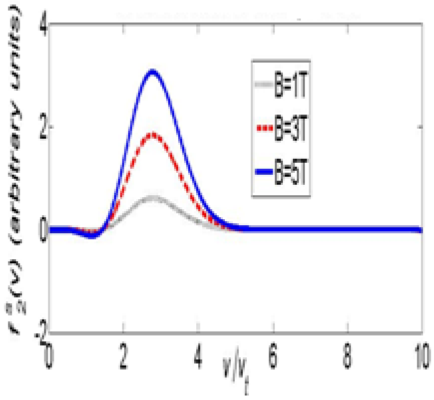

Figure 1. Anisotropic distribution function (arbitrary unit) for different values of the magnetic field

This figure shows that the anisotropy is negative for low velocities ($v \lesssim 2v_t$) which corresponds to a hotter plasma in the parallel direction. However in the high velocity region ($v \gtrsim 2v_t$) the anisotropic component of f is positive and more important. This shows that the fast electrons are in fact responsible for the anisotropy. We present in Fig. 2 the temperature anisotropy as a function of the parameter $\frac{\omega_{ce}}{\nu_{ei}}$.

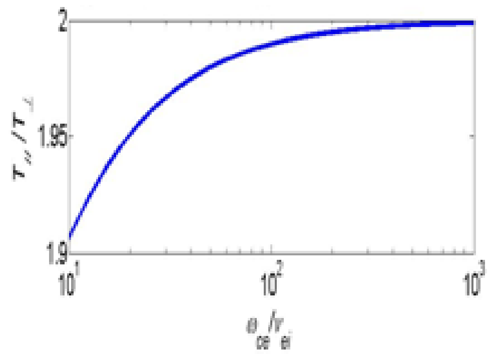

Figure 2. Temperature anisotropy as a function of the rate $\frac{\omega_{ce}}{\nu_{ei}}$

This shows that the anisotropy becomes important as the applied magnetic field becomes intense, and this anisotropy undergoes a saturation in the vicinity of the value 2.

7. Conclusion

To investigate the temperature anisotropy in magnetized plasma we have analytically calculated the distribution function for a highly-magnetized plasma. Using this distribution function, we have calculated the temperature in the parallel and perpendicular directions. We have shown that the temperature is anisotropic and that it is depend on the magnetic field and on the collision frequency. The numerical calculus shows that the anisotropic distribution function is negative in low-velocities region and positive in high-velocity region over a larger band, where the maximum is more important than the minimum. This shows that fast particles are responsible for the temperature anisotropy.

In this study, we have limited the expansion of the distribution function to second order which is sufficient for the study of some physical phenomena occurring in magnetized plasma such as Weibel instability. The plasma is hotter in the parallel direction which can be interpreted by the fact the plasma heating by momentum transfer due to collision is more efficient in the parallel direction. This analytical result could have applications for several physical phenomena occurring in magnetized plasma: the Weibel instability where the growth rate of instability depends on $\frac{T_{\parallel}}{T_{\perp}}$ and the Alfvén wave where dispersion depends on $\frac{T_{\parallel}}{T_{\perp}}$. As an extension to this work, we will calculate the temperature anisotropy for a relativistic magnetized plasma.

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