

# Propagation characteristics of interaction of chirped vector soliton trains in birefringent optical fibers with variable coefficients in the presence of third and fourth order dispersion and quintic nonlinearity

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## **Abstract**

*This paper presents a numerical study of propagation characteristics of chirped vector soliton trains in birefringent optical fibers with variable coefficients. This study is done using the compact split step Padé scheme (CSSPS), in the presence of third and fourth order dispersion and quintic nonlinearity. We are interested in the interaction of adjacent vector solitons in managed birefringent fibers by changing different parameters such as the chirp and the distance between two adjacent vector solitons. In all cases, the energy of each chirped vector soliton remains conserved.*

*Keywords: Vector soliton trains; Chirped solitons; Birefringent optical fibers; Compact split step Padé scheme; Temporal waveform*

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## **1. Introduction**

Since the first observation of optical soliton in fused silica fibers by Hasegawa and Tappert [1,2] in 1973, it has demonstrated a big interest because of its possibility to propagate through considerable distances without distortion. Since then, a lot of theoretical and experimental work has been done on the optical soliton propagation in optical fibers [1-20]. In single mode fibers, the pulse propagation in a nonlinear optical fiber can be described by the nonlinear Schrödinger (NLS) equation [1,2,3].

In optical fibers, birefringence is a natural phenomenon that takes place because of effects due to different elements such as curves, turns and anisotropic stress of fibers due to bending [20]. These effects lead up to differential group delay.

Menyuk is the first which has studied the effect of birefringence on the pulse propagation in optical fiber [16]. He predicted that optical vector solitons can be stable under certain operating conditions [17,18]. The two orthogonal polarization modes of the fiber can be coupled together through nonlinear effects leading to cross phase modulation. The propagation of vector solitons in birefringent fibers is governed by a system of two coupled nonlinear Schrödinger (CNLS) equations. The CNLS equations have been the subject of concentrated theoretical, experimental and numerical investigations during recent years [1,2].

Optical vector solitons [1,2] have a rich nonlinear dynamics due to their wide range of applications in fiber-optic-based communication systems [9], and their multi-component structure [4].

In general, adjacent scalar solitons can interact with each other in dispersion managed fibers leading to rich dynamics, see for example reference [19]. The birefringence in fibers affects significantly the attractive force and leads to collisions of two in-phase solitons in isotropic fibers [21]. In this study, we are interested in the interaction of adjacent vector solitons in managed birefringent fibers. The propagation of vector solitons in managed birefringent optical fibers is governed by the two coupled NLS equations [1,2,4,6,8] with variable coefficients [13]. This kind of equations is not integrable except in few particular cases. We use the numerical compact split step Padé scheme (CSSPS) [5] to investigate the interaction of vector solitons by changing different parameters such as the chirp and the distance between two adjacent vector solitons.

This paper is organized as follows. In Section 2, the coupled nonlinear Schrödinger (CNLS) equations with variable coefficients are mentioned in the presence of third and fourth order dispersion and quintic nonlinearity [10,11,12,14]. In Section 3, we present our numerical model and set the initial conditions with linear chirp. Results and discussions are shown in Section 4.

## 2. Theoretical model

Managed vector solitons are governed by the coupled nonlinear Schrödinger (CNLS) equations with variable coefficients [1,2,4,5,6,8]:

$$\begin{aligned} \frac{\partial}{\partial z} \begin{pmatrix} u \\ v \end{pmatrix} + \delta \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{\partial}{\partial t} \begin{pmatrix} u \\ v \end{pmatrix} + \frac{i}{2} d(z) \frac{\partial^2}{\partial t^2} \begin{pmatrix} u \\ v \end{pmatrix} - \\ i \sum_{n=3}^{\infty} \frac{i^n}{n!} \beta_n \frac{\partial^n}{\partial t^n} \begin{pmatrix} u \\ v \end{pmatrix} + \Gamma(z) \begin{pmatrix} u \\ v \end{pmatrix} = \\ i\gamma(z) \begin{pmatrix} |u|^2 + \varepsilon|v|^2 & 0 \\ 0 & \varepsilon|u|^2 + |v|^2 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \\ ik \begin{pmatrix} |u|^4 + \varepsilon|v|^4 & 0 \\ 0 & \varepsilon|u|^4 + |v|^4 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}, \quad (1) \end{aligned}$$

Where  $u(z, t)$  and  $v(z, t)$  are the slowly varying amplitudes of the two polarization modes in the fiber.  $z$  and  $t$  are respectively the normalized distance and time.  $\delta$  is the difference of the group velocities between the two polarization components.  $\varepsilon$  is the cross-phase modulation (XPM) coefficient. For linearly birefringent fibers,  $\varepsilon = 2/3$ .  $k$  is the quintic nonlinearity coefficient.  $\beta_3$  and  $\beta_4$  are respectively the third and fourth order dispersion. The functions  $d(z)$ ,  $\gamma(z)$  and  $\Gamma(z)$  are respectively the managed group velocity dispersion (GVD), the managed self-phase modulation (SPM) and gain (or loss), They are given by the following expressions[7]:

- the varying GVD parameter

$$d(z) = \exp(\sigma z) \gamma(z) / d_0, \quad (2)$$

- the nonlinear parameter

$$\gamma(z) = \gamma_0 + \gamma_1 \sin(gz), \quad (3)$$

- and the gain (or loss) distributed parameter

$$\Gamma(z) = \sigma / 2, \quad (4)$$

Where  $d_0$  is a parameter related to the initial peak power,  $\sigma$  is a parameter describing gain or loss, and  $\gamma_0$ ,  $\gamma_1$  and  $g$  are the parameters describing Kerr nonlinearity. For our study, we take the parameters  $d_0 = 1$ ,  $\gamma_0 = 0$ ,  $\gamma_1 = 1$ ,  $g = 1$  and  $\sigma = 0$  [7].

## 3. Initial condition

In order to do our numerical simulations, we choose the following initial conditions [7]:

$$u(0, t) = \cos\alpha \exp(-iCt^2/2) / \cosh(t), \quad (5)$$

$$v(0, t) = \sin\alpha \exp(-iCt^2/2) / \cosh(t), \quad (6)$$

Where,  $C$  is the linear chirp parameter and the angle  $\alpha$  representing the polarization angle.

## 4. Results and discussions

In this section, we will be interested in the interaction of adjacent vector solitons in managed birefringent fibers. But first, let us find the shape of the envelope of the scalar soliton in the absence of any perturbation in section 4. 1 with  $\alpha = 0^\circ$ .

Then, when  $\alpha = 45^\circ$ , the two polarization components of a vector soliton have the same amplitude. The section 4.2 addresses the propagation of chirped vector soliton in a managed optical fiber and their interaction in the presence of third and fourth order dispersion and quintic nonlinearity.

### 4.1. First case: $\alpha = 0^\circ$

When  $\alpha = 0^\circ$ , this is the case of an optical soliton with a polarization parallel to the fiber axis. Scalar optical solitons have played an important role in the field of high-bit data transmission systems because of the unique properties. They can propagate over long distances keeping their shape unchanged. This property is due to the balance between the dispersion and the nonlinear effects.

Fig. 1 shows the propagation of optical soliton in the absence of any perturbation. As it is well known, optical soliton maintains its shape undistorted.

When we take into account the chirp, we can note that a negative chirp makes the soliton broadening, while; a positive chirp leads to a soliton compression (Fig. 2).

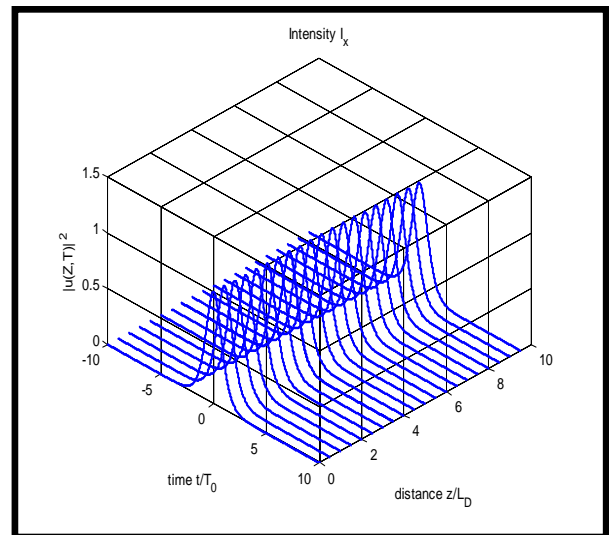


Figure 1. Evolution of unchirped scalar soliton in managed optical fiber with the parameters  $\alpha = 0^\circ$ ,  $C = 0$  and  $\sigma = 0$

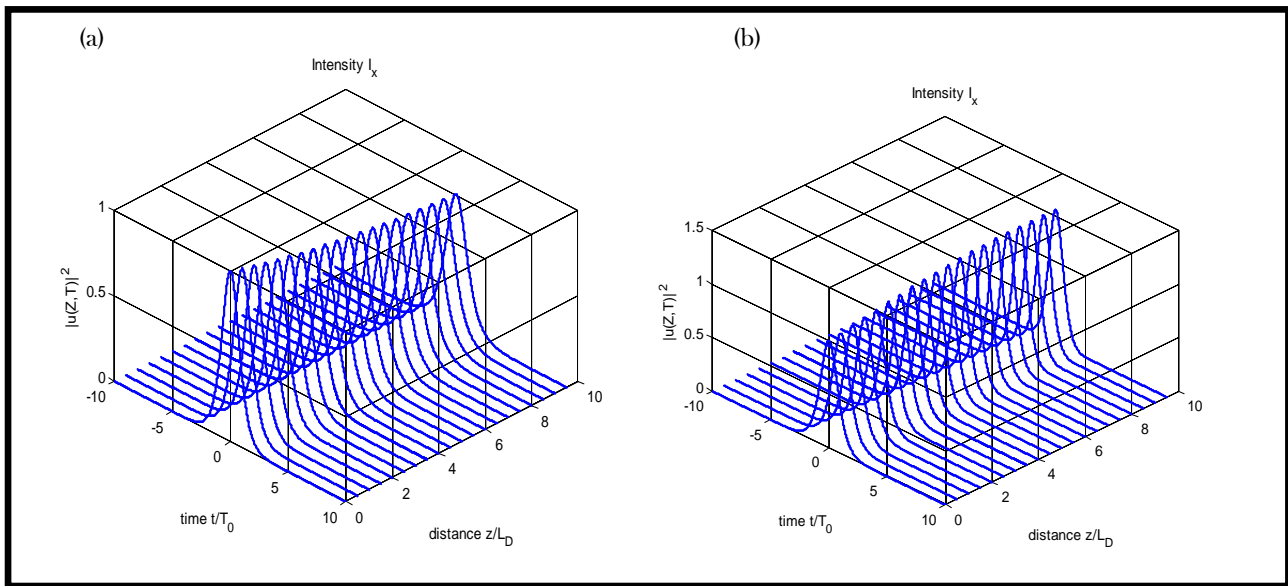


Figure 2. Evolution of chirped scalar soliton in managed optical fiber with the parameters  $\alpha = 0^\circ$ , and  $\sigma = 0$ :  
 (a) with  $C = -0.2$ , and (b) with  $C = 0.2$

#### 4.2. Second case: $\alpha = 45^\circ$

In the following three cases, we examine the evolution of vector soliton trains in a managed optical fiber under the effect of the chirp, the quintic nonlinearity, the third and fourth order dispersion when  $\alpha = 45^\circ$  (the two polarization components have the same amplitude).

##### 4.2.1. Chirped vector soliton in a managed optical fiber

First, we begin with the simple case where we will examine the evolution of vector soliton in a managed optical fiber, under the effect of the chirp. The profile of the intensity of the two polarization components of chirped vector soliton in managed optical fiber are depicted in Fig. 3 ( $C < 0$ ) and Fig. 4 ( $C > 0$ ). As in the case of scalar chirped soliton, a negative chirp leads to the optical vector soliton broadening (Fig. 3), while; a positive chirp leads to a vector soliton compression (Fig. 4) with the increase of propagation distance.

To study the effect of third order dispersion alone,  $\beta_4$  and  $k$  are set to zero in equation (1). Fig. 5 shows the evolution of the two polarization components of the chirped vector soliton in a managed optical fiber for the following parameters:  $\alpha = 45^\circ$ ,  $C = 0.2$ ,  $\beta_3 = 1$ ,  $\beta_4 = 0$ ,  $k = 0$  and  $\sigma = 0$ .

We note that the pulse width of the two polarization components of the chirped vector soliton in a managed optical fiber increases along the propagation distance. At the same time, there is an asymmetric oscillation on the

trailing edge and a shift of the center of the two polarization components of the chirped vector soliton.

The evolution of the two polarization components of the chirped vector soliton in a managed optical fiber under the effect of fourth order dispersion is studied for the following parameters  $\alpha = 45^\circ$ ,  $C = 0.2$ ,  $\beta_3 = 0$ ,  $k = 0$  and  $\sigma = 0$ . From the plots, we found that a higher value of fourth order dispersion broadens the pulse width as shown in Fig. 6 with the parameter  $\beta_4 = 0.08$ , and lower values of the same reduce the pulse width as shown in Fig. 7 with the parameter  $\beta_4 = 0.01$ , which, in turn, will be practical in optical communications.

From Fig. 8 with the parameters  $\alpha = 45^\circ$ ,  $C = 0.2$ ,  $\beta_3 = 0$ ,  $\beta_4 = 0$ ,  $k = -0.05$  and  $\sigma = 0$ , it is clear that the role of quintic nonlinearity on the propagation characteristics of the two polarization components of the chirped vector soliton in a managed optical fiber is unimportant.

We study now the propagation of the two polarization components of chirped vector soliton in birefringent optical fiber under the combined action of third order dispersion, the fourth order dispersion and the quintic nonlinearity. We note that, in addition to the asymmetric oscillation on the trailing edge, the increase of the pulse width and the shift of the center of the two polarization components of chirped vector soliton along the propagation distance; there is also an oscillation in the amplitude (maximum) of the two polarization components of the chirped vector soliton in a managed optical fiber (Fig.9).

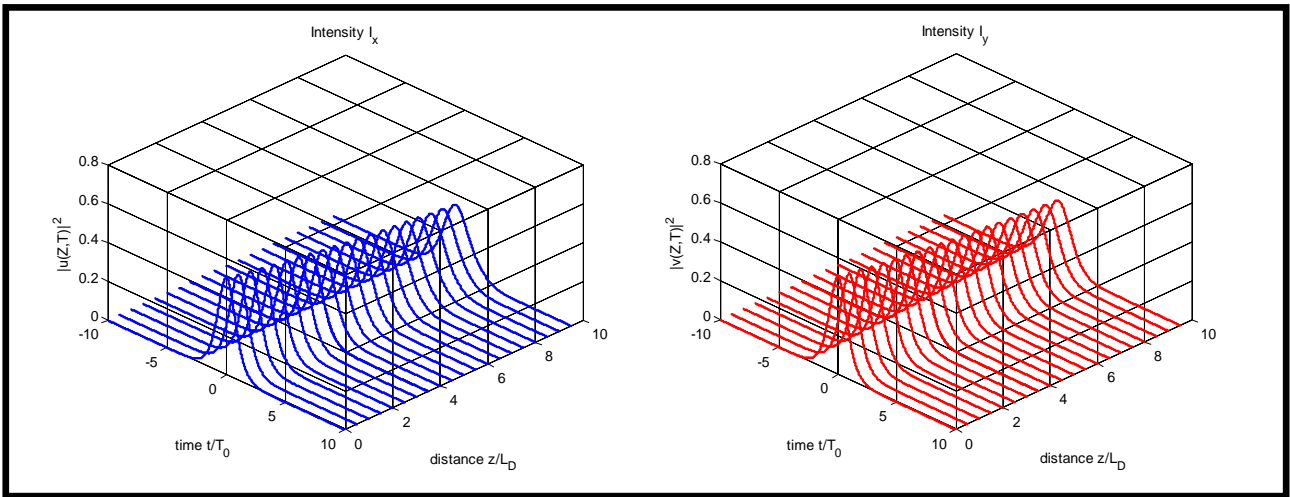


Figure 3. Evolution of the two polarization components of chirped vector soliton in birefringent managed optical fiber with the parameters  $\alpha = 45^\circ$ ,  $C = -0.2$  and  $\sigma = 0$

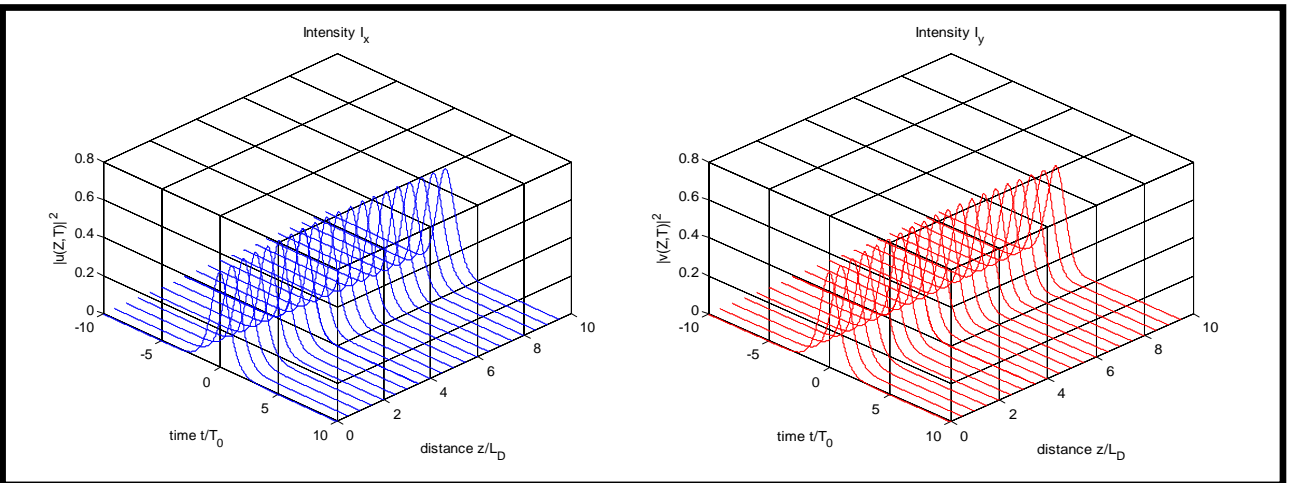


Figure 4. Evolution of the two polarization components of chirped vector soliton in birefringent managed optical fiber with the parameters  $\alpha = 45^\circ$ ,  $C = 0.2$  and  $\sigma = 0$

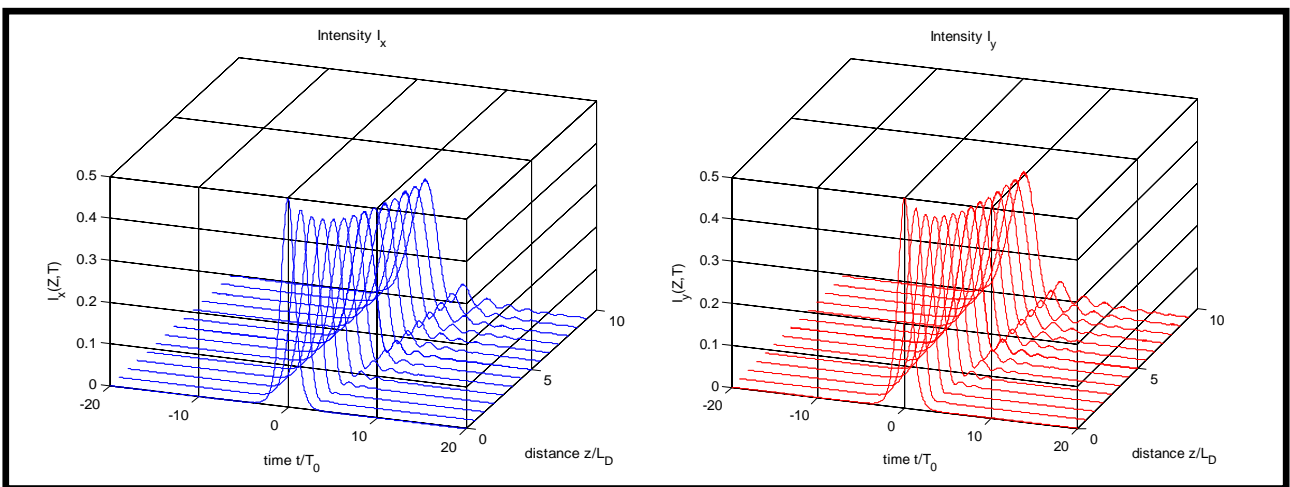


Figure 5. Evolution of the two polarization components of chirped vector soliton in birefringent managed optical fiber with the parameters  $\alpha = 45^\circ$ ,  $C = 0.2$ ,  $\beta_3 = 1$ ,  $\beta_4 = 0$ ,  $k = 0$  and  $\sigma = 0$

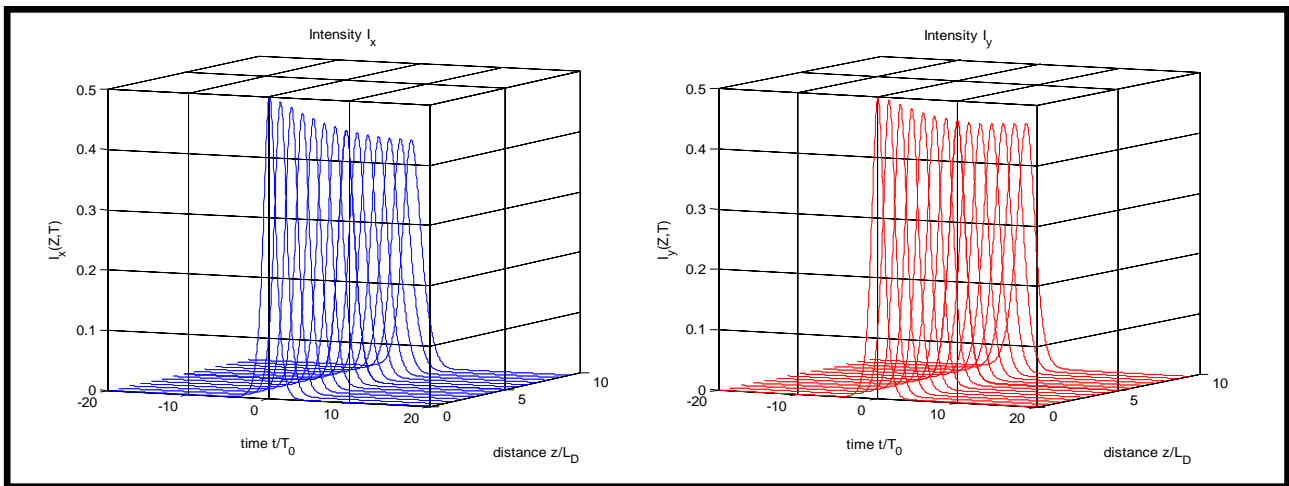


Figure 6. Evolution of the two polarization components of chirped vector soliton in birefringent managed optical fiber with the parameters  $\alpha = 45^\circ$ ,  $C = 0.2$ ,  $\beta_3 = 0$ ,  $\beta_4 = 0.08$ ,  $k = 0$  and  $\sigma = 0$

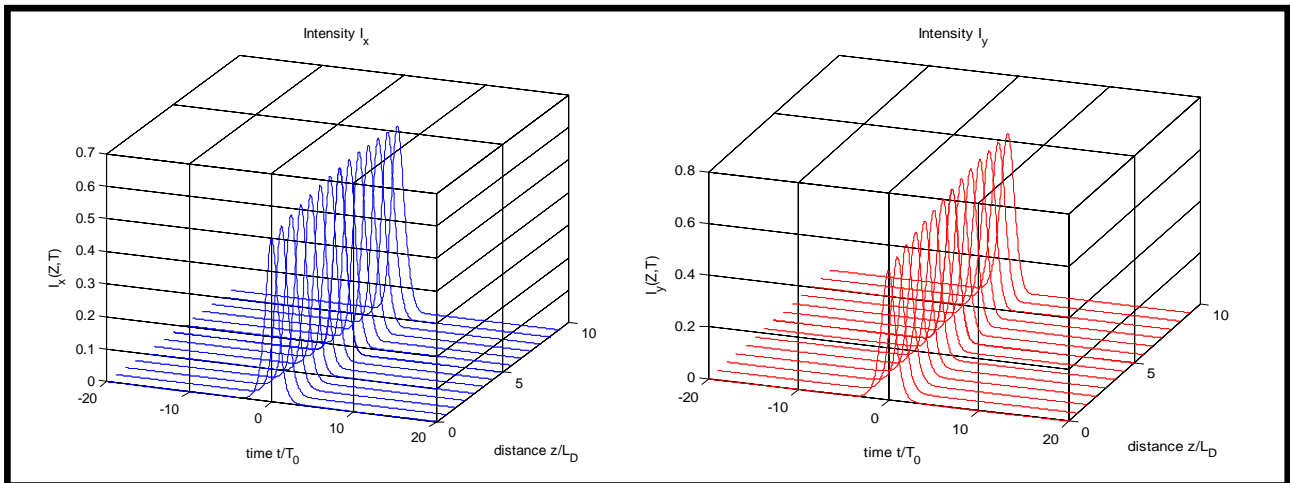


Figure 7. Evolution of the two polarization components of chirped vector soliton in birefringent managed optical fiber with the parameters  $\alpha = 45^\circ$ ,  $C = 0.2$ ,  $\beta_3 = 0$ ,  $\beta_4 = 0.01$ ,  $k = 0$  and  $\sigma = 0$

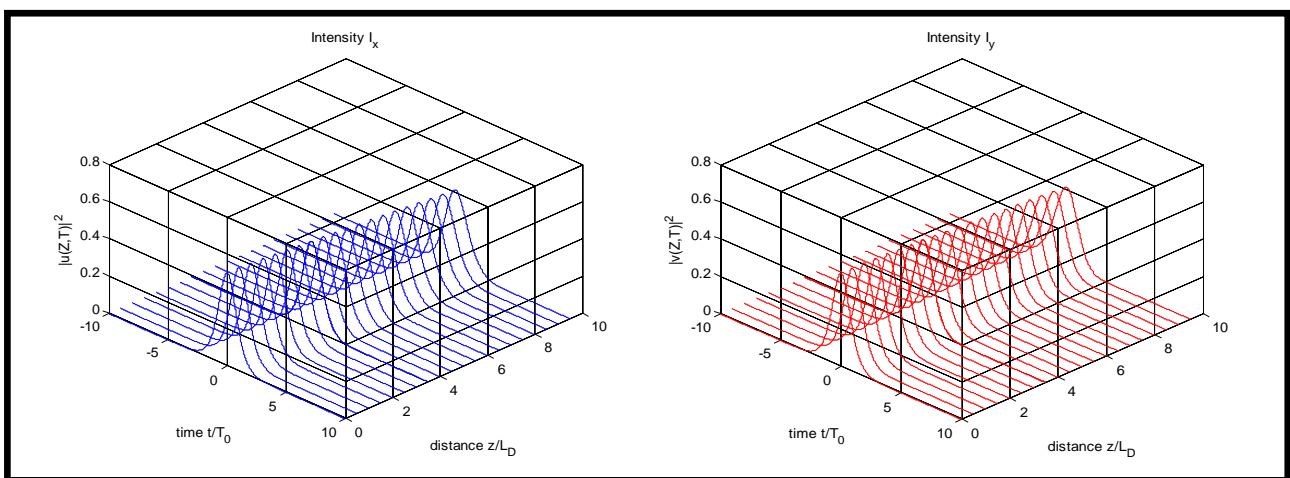


Figure 8. Evolution of the two polarization components of chirped vector soliton in birefringent managed optical fiber with the parameters  $\alpha = 45^\circ$ ,  $C = 0.2$ ,  $\beta_3 = 0$ ,  $\beta_4 = 0$ ,  $k = -0.05$  and  $\sigma = 0$

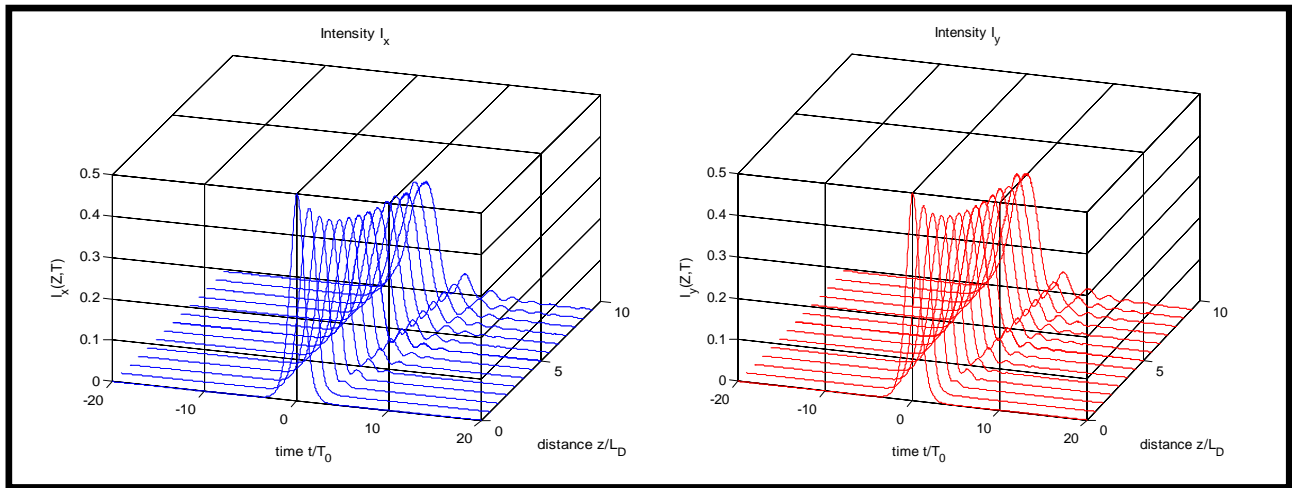


Figure 9. Evolution of the two polarization components of chirped vector soliton in birefringent managed optical fiber with the parameters  $\alpha = 45^\circ, C = -0.2, \beta_3 = 1, \beta_4 = 0.01, k = -0.05$  and  $\sigma = 0$

4.2.2. Interaction between two chirped vector solitons in a managed optical fiber

In this case, we examine the effect of the chirp  $C$  and the distance  $d$  between two adjacent vector solitons on the interaction between two vector solitons in managed optical fibers.

Whatever the value of the distance  $d$ , a negative chirp makes an increase of the amplitude with a decrease of the width of the first vector soliton, while the inverse happens for the second one. A positive chirp makes a decrease of the amplitude with an increase of the width of the first vector soliton, while the inverse happens for the second one. This case is the inverse of the one with a negative chirp.

In the case of  $C = 0$ , the propagation of the two components of interaction of two vector solitons in birefringent optical fiber is affected by the variation of the distance between two adjacent vector solitons. We note a decrease of the amplitude with an increase of the width of the two polarization components of interaction of two unchirped vector solitons in birefringent optical fiber.

If we take  $d = 4$ , we note from Fig. 10, Fig.11 and Fig. 12 that the propagation of two polarization components of interaction of two vector solitons in birefringent optical fiber is affected by the presence of the chirp. If we decrease the distance between two vector solitons ( $d = 2$ ), we note in Fig. 13, some oscillations between the two vector solitons. This oscillations increase with the decrease of the distance  $d$  like in Fig. 14 with  $d = 1$ .

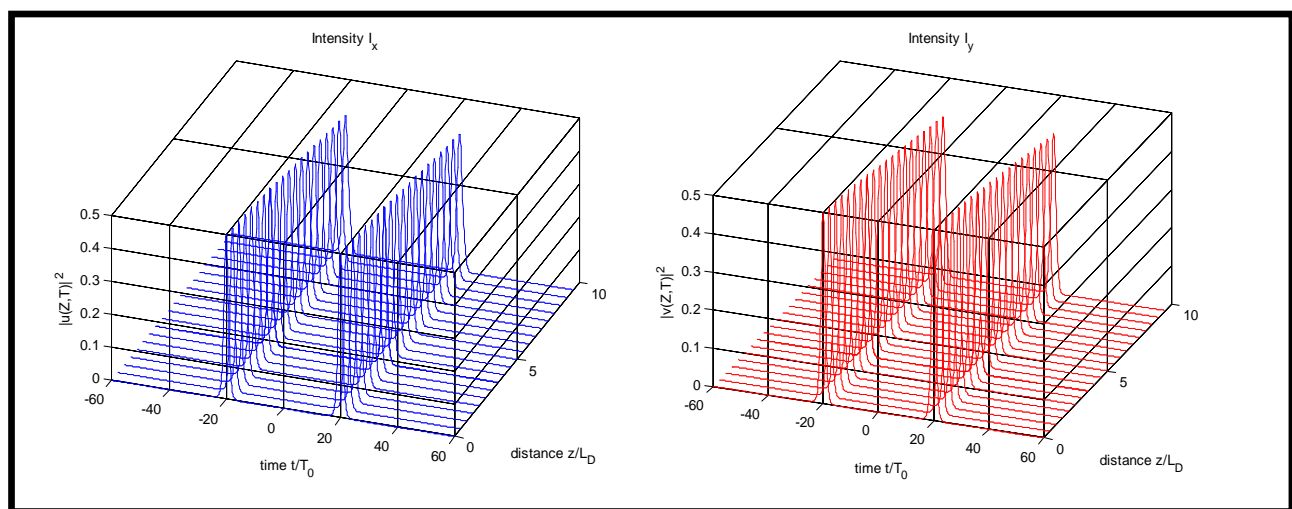


Figure10. Evolution of the two polarization components of interaction of two unchirped vector solitons in birefringent optical fiber with the parameters  $\alpha = 45^\circ, C = 0, \sigma = 0$ , and  $d = 4$

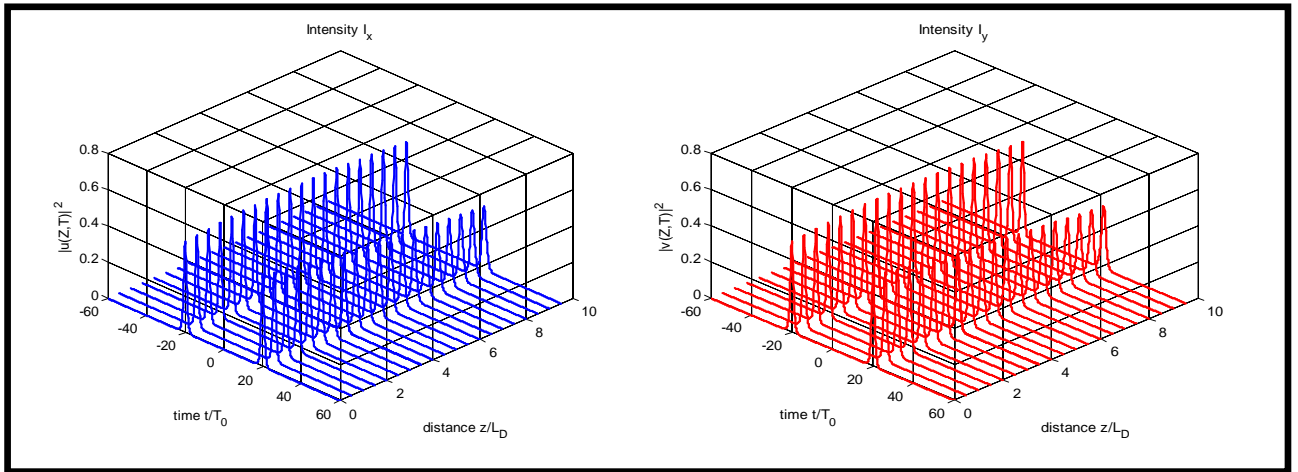


Figure 11. Evolution of the two polarization components of interaction of two chirped vector solitons in birefringent optical fiber with the parameters  $\alpha = 45^\circ$ ,  $C = -0.2$ ,  $\sigma = 0$ , and  $d = 4$

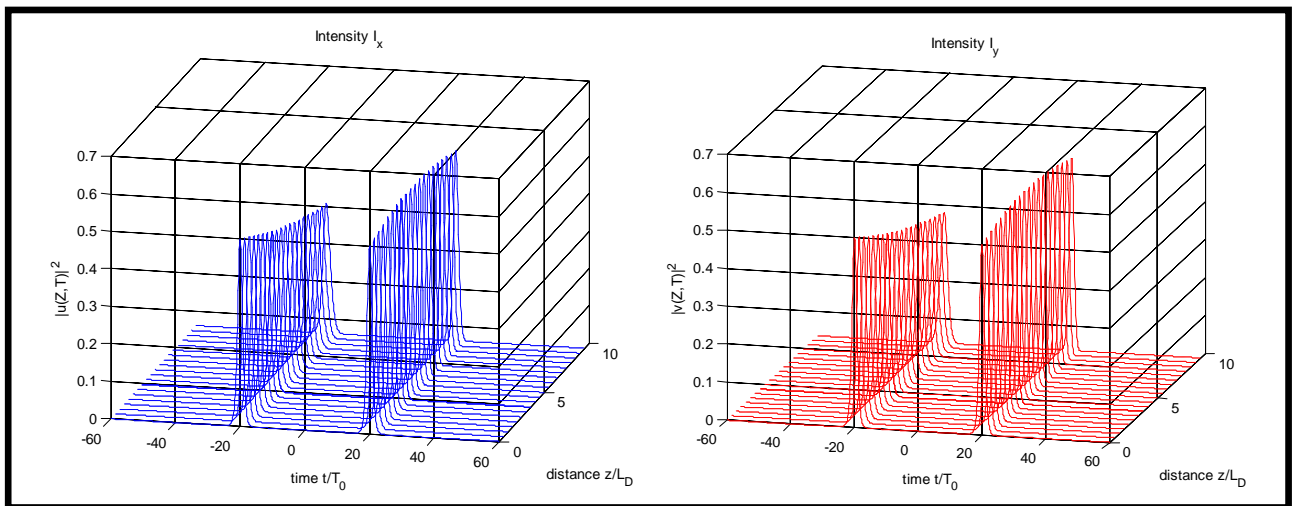


Figure 12. Evolution of the two polarization components of interaction of two chirped vector solitons in birefringent optical fiber with the parameters  $\alpha = 45^\circ$ ,  $C = 0.2$ ,  $\sigma = 0$ , and  $d = 4$

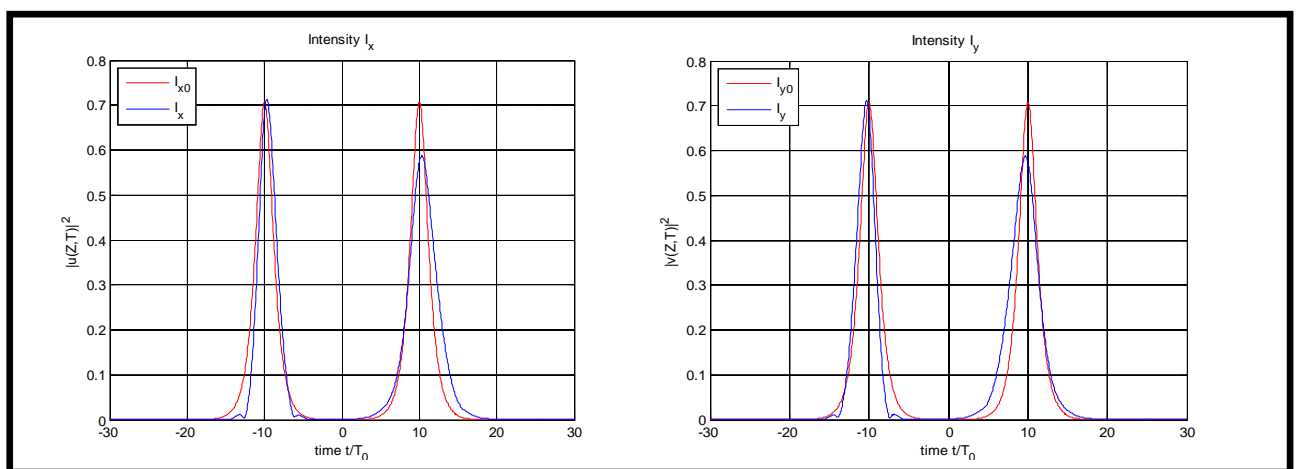


Figure 13. Evolution of the two polarization components of interaction of two chirped vector solitons in birefringent optical fiber with the parameters  $\alpha = 45^\circ$ ,  $C = -0.2$ ,  $\sigma = 0$ , and  $d = 2$

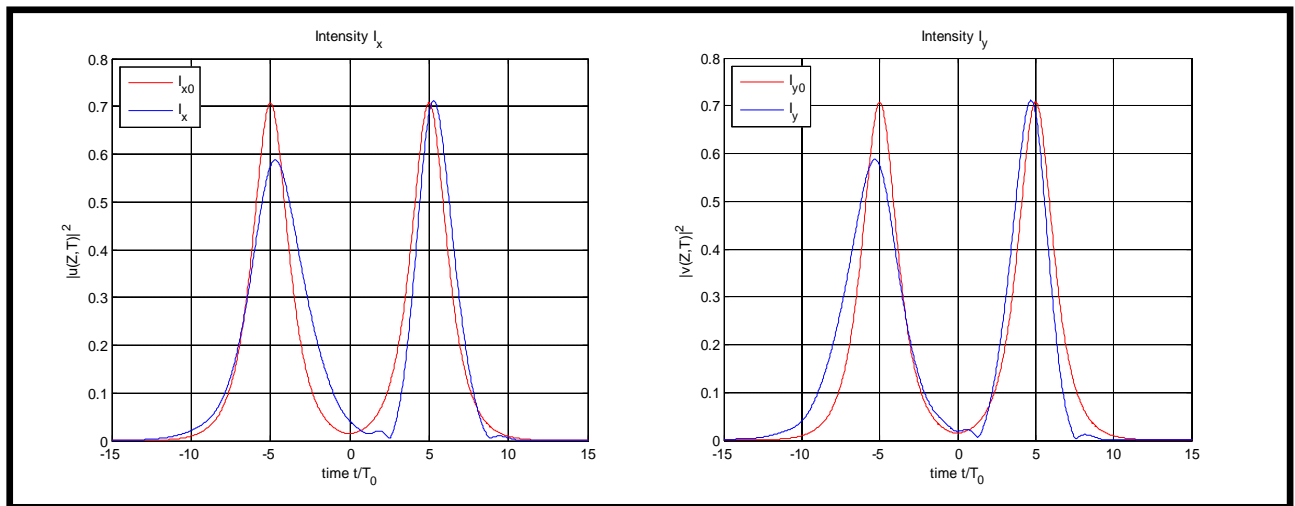


Figure 14. Evolution of the two polarization components of interaction of two chirped vector solitons in birefringent optical fiber with the parameters  $\alpha = 45^\circ$ ,  $C = 0.2$ ,  $\sigma = 0$ , and  $d = 1$

4.2.3. Interaction between three chirped vector solitons in a managed optical fiber

In this section, we examine the effect of the chirp  $C$  and the distance  $d$  between two adjacent vector solitons  $d$  on the propagation of the interaction between three vector solitons in managed optical fibers.

In the case of negative chirp, we note from Fig. 15 an increase in the amplitude and a decrease of the width of the second vector soliton, while the inverse is happen to the first and third ones. In figure 16, a positive chirp makes a decrease of the amplitude with an increase of the width of the second vector soliton, while the inverse happens for the first and third ones. This case is the inverse of the one with negative chirp (Figure 15). In all this cases the energy of each soliton is conserved.

5. Conclusion

In this paper, the propagation characteristics of chirped vector solitons trains in managed birefringent optical fibers are studied numerically. This study is done using the compact split step Padé scheme (CSSPS). The propagation of vector solitons has a rich dynamics. A negative chirp makes the chirped vector soliton broadening, while; a positive chirp leads to a chirped vector soliton compression. From plot, it is clearly noted that, the quintic nonlinearity has a marginal role on the propagation characteristics of the two components of the chirped vector soliton. The evolution of chirped vector soliton trains in managed optical fiber is submitted not only to the presence of the effect of chirp, but also to the interaction between the adjacent vector solitons. In all cases, the energy of each chirped vector soliton remains conserved.

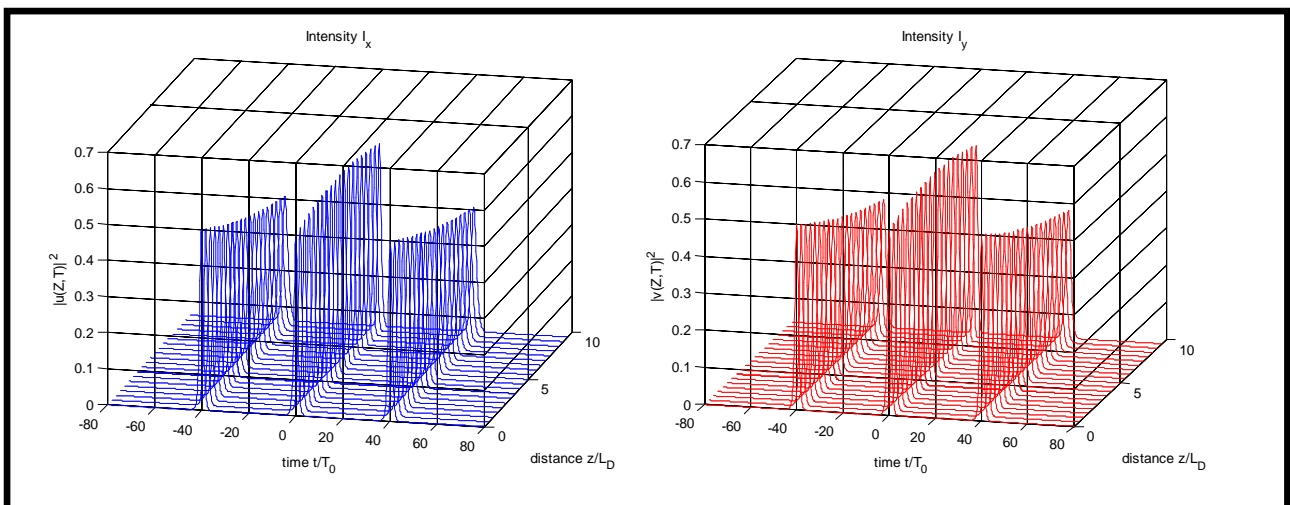


Figure 15. Evolution of the two polarization components of interaction of three chirped vector solitons in birefringent optical fiber with the parameters  $\alpha = 45^\circ$ ,  $C = -0.2$ ,  $\sigma = 0$ , and  $d = 2$



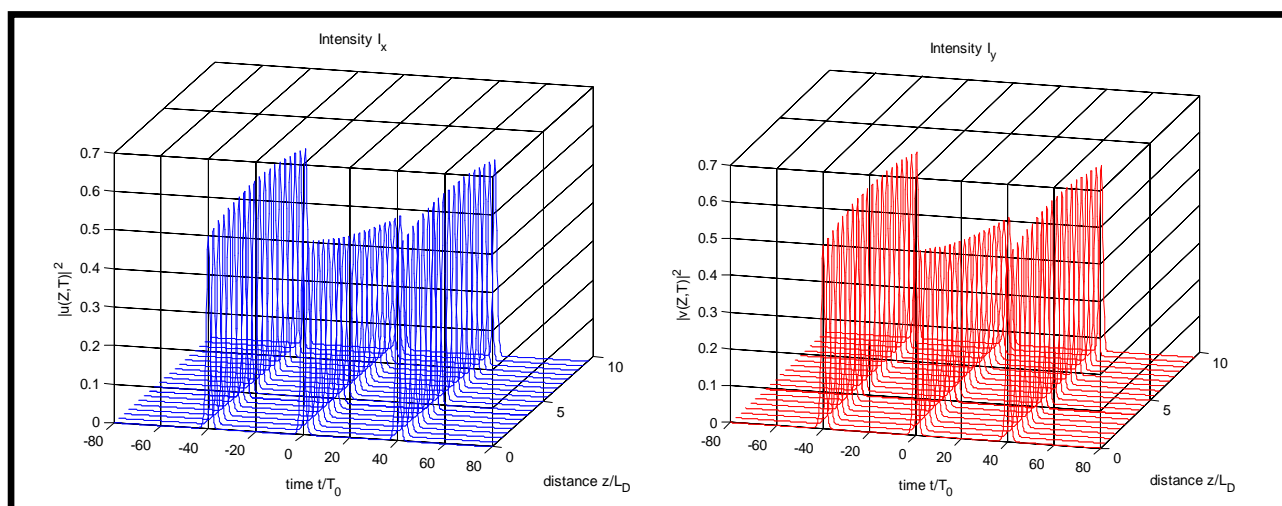


Figure16. Evolution of the two c polarization components of interaction of three chirped vector solitons in birefringent optical fiber with the parameters  $\alpha = 45^\circ$ ,  $C = 0.2$ ,  $\sigma = 0$ , and  $d = 2$

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